Note Title 4/23/2019

$$\frac{dy(t)}{dt} + a\,y(t) = b\,x(t)$$

$$\downarrow$$

$$sY(s) - y(0^-) + aY(s) = bX(s)$$

$$Y(s)[s+a] = y(0^-) + bX(s) \rightarrow \text{Transfer function } H(s)$$

$$Y(s) = \frac{y(0^-)}{s+a} + \frac{b}{s+a}X(s)$$

$\frac{b}{s+a}X(s)$: Response due to $X(s)$

$\frac{y(0^-)}{s+a}$: Response due to initial conditions

$$\dot{y}(t) = \frac{dy(t)}{dt}$$

$$\ddot{y}(t) = \frac{d^2y(t)}{dt^2}$$

$$\therefore H(s) = \frac{b}{s+a}$$

$$\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0\,y(t) = b_1\frac{dx(t)}{dt} + b_0\,x(t)$$

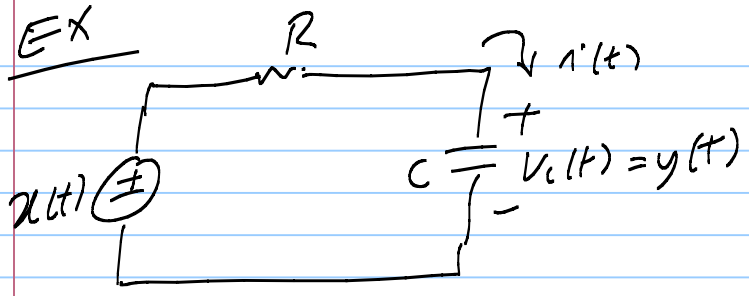
$$s^2Y(s) - sy(0^-) - \dot{y}(0^-) + a_1[sY(s) - y(0^-)] + a_0Y(s) =$$

$$b_1sX(s) - b_0X(s)$$

$$Y(s) = \frac{sy(0^-) + \dot{y}(0^-) + a_1y(0^-)}{s^2 + a_1s + a_0} + \frac{b_1s + b_0}{s^2 + a_1s + a_0}X(s)$$

$$H(s) = \frac{b_1s + b_0}{s^2 + a_1s + a_0}$$

EX



Find $H(s)$

$$i(t) = C\frac{dv_c(t)}{dt} = C\frac{dy(t)}{dt}$$

Solution 1

$$-x(t) + Ri(t) + y(t) = 0$$

$$-x(t) + RC\frac{dy(t)}{dt} + y(t) = 0$$

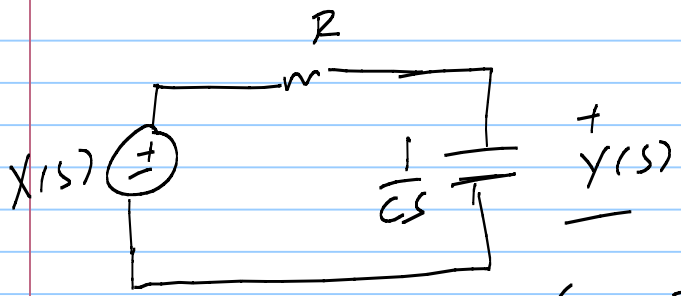
To Find $H(s)$, All I.C. $\equiv 0$

$$-X(s) + RCsY(s) + Y(s) = 0$$

$$Y(s) = \frac{1}{RCs+1}X(s)$$

$$\text{or } H(s) = \frac{1}{RCs+1} = \frac{1/RC}{1/RC + s}$$

Solution 2



$$Y(s) = X(s)\left(\frac{\frac{1}{Cs}}{R + \frac{1}{Cs}}\right) = X(s)\left(\frac{1}{RCs+1}\right)$$

$$\text{or } \boxed{H(s) = \frac{1}{RCs+1}}$$

EX

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

$$H(s) = \frac{2}{s^2+6s+8}$$

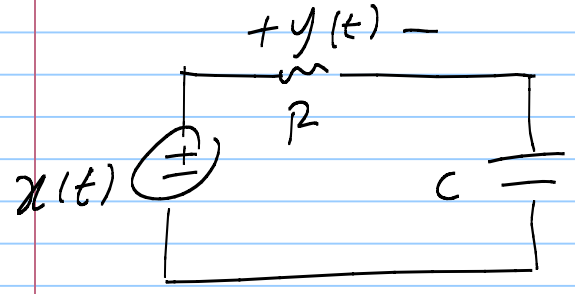
OR I.C. $\equiv 0$

$$s^2Y(s) + 6sY(s) + 8Y(s) = 2X(s)$$

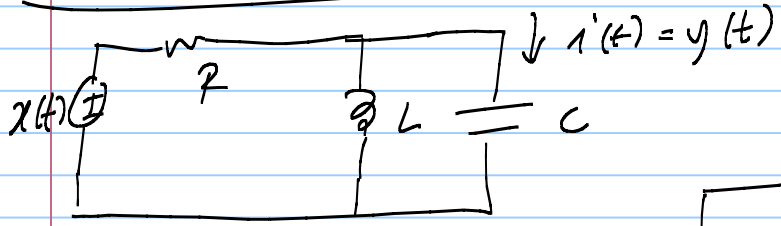
$$Y(s)[s^2+6s+8] = 2X(s)$$

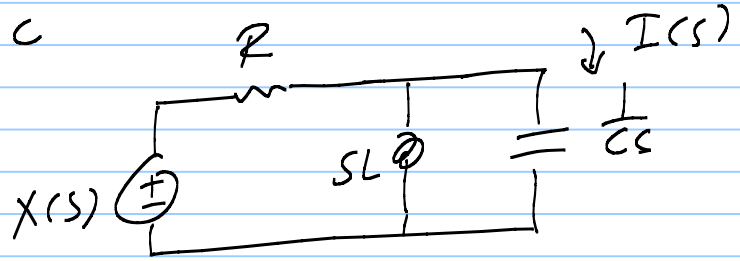
$$H(s) = \frac{Y(s)}{X(s)} = \frac{2}{s^2+6s+8}$$

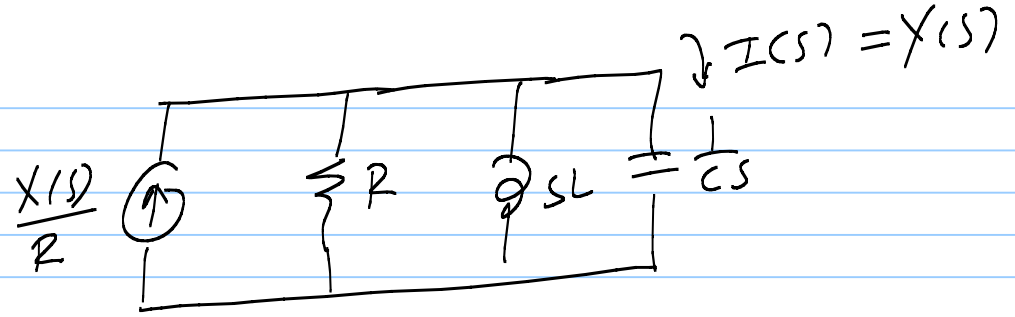
Class Activity



Find $H(s) = \dfrac{RCs}{RCs+1} = \dfrac{s}{s+1/RC}$



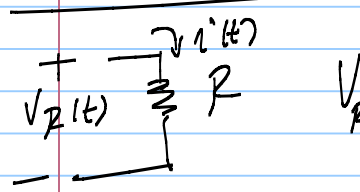




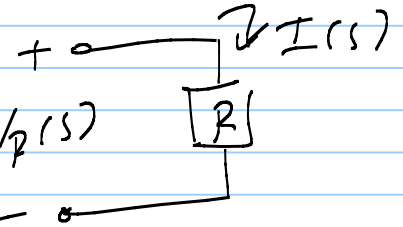
$$Y(s) = \frac{X(s)}{R}\left(\frac{1/(1/Cs)}{\frac{1}{R}+\frac{1}{SL}+\frac{1}{1/Cs}}\right) = \frac{X(s)}{R}\left(\frac{Cs}{\frac{1}{R}+\frac{1}{Ls}+Cs}\right)$$

$$\frac{Y(s)}{X(s)} = \frac{Cs}{1+\frac{R}{Ls}+RCs} = \frac{LCs^2}{Ls+R+RLCs^2}$$

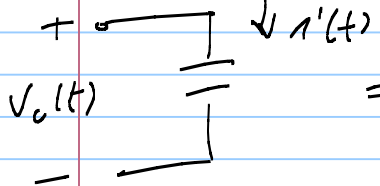
$$= \frac{\frac{1}{R}s^2}{s^2+\frac{1}{RC}s+\frac{1}{LC}}$$



$V_R(s) = RI(s)$



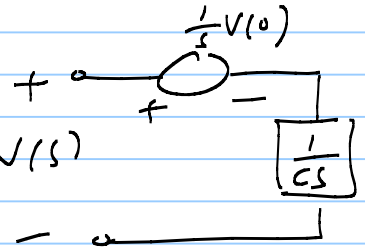
$$i_c(t) = C\frac{dv_c(t)}{dt}$$

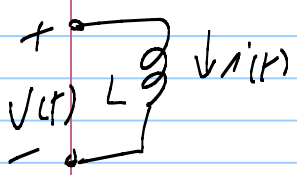


$$\Rightarrow I(s) = CsV(s) - Cv(0)$$

or $sV(s) - v(0) = \frac{1}{C}I(s)$

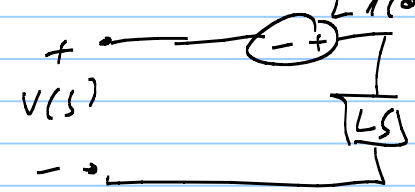
$$\hookrightarrow V(s) - \frac{1}{s}v(0) = \frac{1}{Cs}I(s)$$

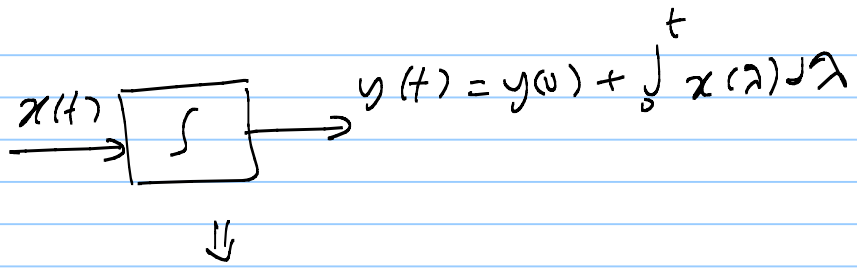


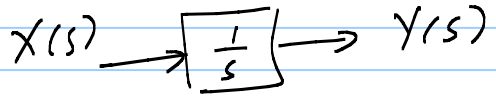


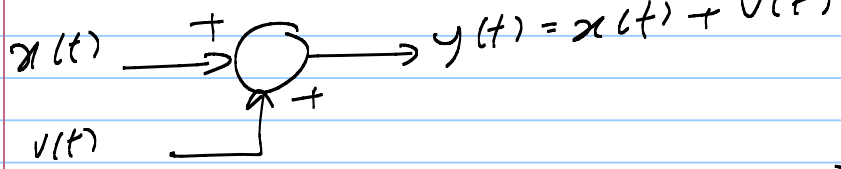
$$V_L(t) = L\frac{di(t)}{dt}$$

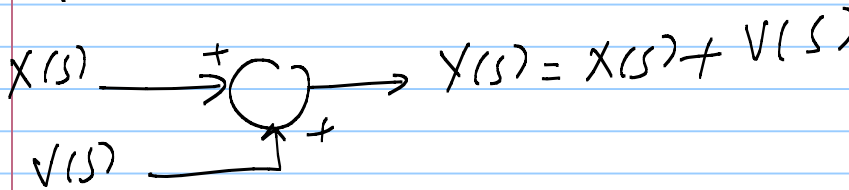
$$V(s) = LsI(s) - Li(0)$$

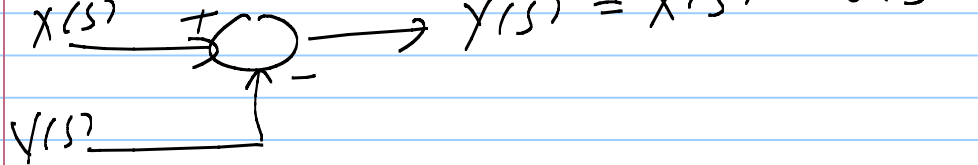


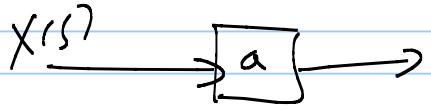


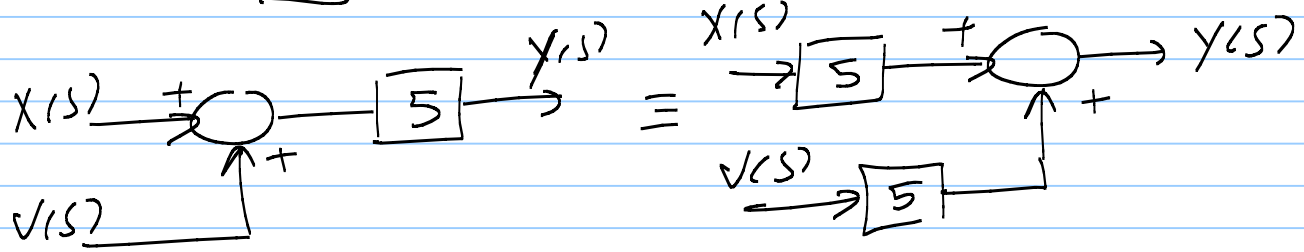


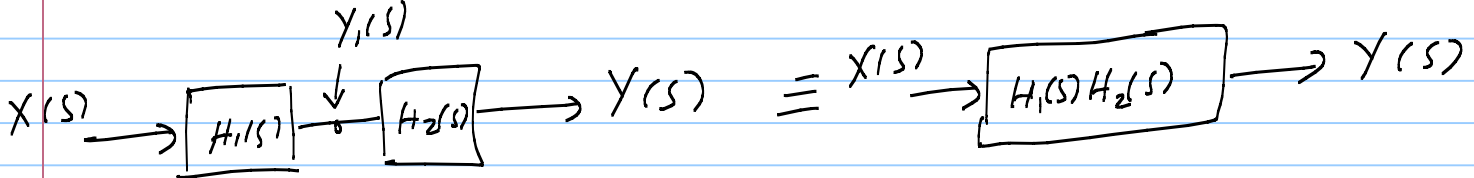




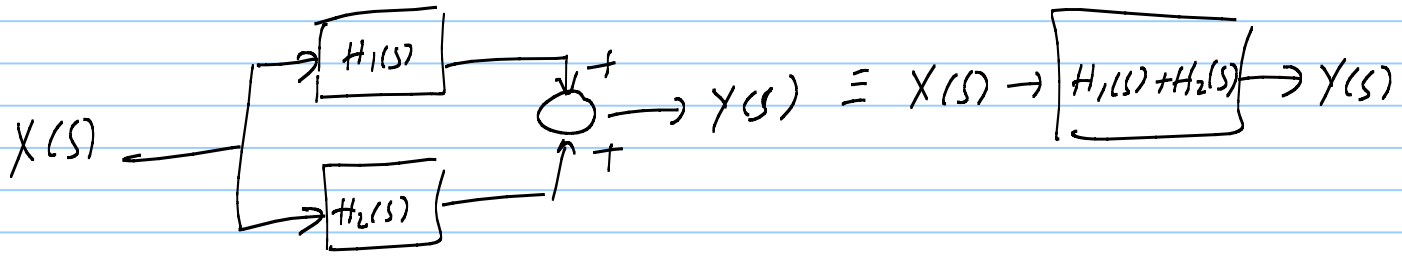






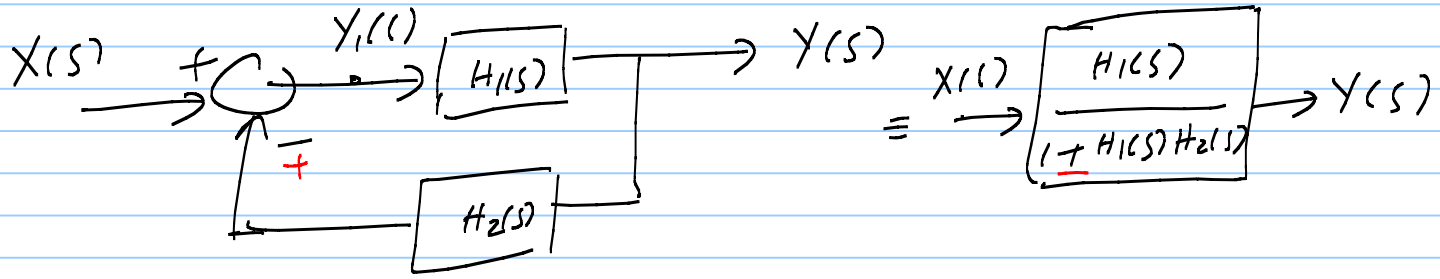


$$Y(s) = Y_1(s)\,H_2(s)$$
$$Y_1(s) = X(s)\,H_1(s)$$
$$Y(s) = H_1(s)H_2(s)\,X(s)$$



$$Y(s) = X(s)\,H_1(s) + X(s)\,H_2(s)$$

$$= X(s)\left(H_1(s) + H_2(s)\right)$$



Negative Feedback

$$Y_1(s) = X(s) - Y(s)\,H_2(s)$$

$$Y(s) = Y_1(s)\,H_1(s)$$

$$Y(s) = X(s)\,H_1(s) - Y(s)\,H_1(s)\,H_2(s)$$

$$Y(s) + Y(s)\,H_1(s)\,H_2(s) = X(s)\,H_1(s)$$

$$Y(s)\left[1 + H_1(s)\,H_2(s)\right] = X(s)\,H_1(s)$$

$$\boxed{\frac{Y(s)}{X(s)} = \frac{H_1(s)}{1 + H_1(s)\,H_2(s)}}$$